Laplace transform

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Applications of Laplace transform in differential equations

To solve the linear differential equations under specified conditions, the formula for the Laplace transforms of the derivation of a function and the process of finding the inverse transform play a key role in this application.

We are using the following formulas:-

(i)$\mathrm{L}\{y'(t)\} = \mathrm{SL}\{\mathrm{y(t)}\} - \mathrm{y}(0)$

(ii)$\mathrm{L}\{y''(t)\} = \mathrm{S}^2\mathrm{L}\{\mathrm{y(t)}\} - \mathrm{Sy}(0) - \mathrm{y}'(0)$

(iii)$\mathrm{L}\{y'''(t)\} = \mathrm{S}^3\mathrm{L}\{\mathrm{y(t)}\} - \mathrm{S}^2\mathrm{y}(0) - \mathrm{Sy}''(0) - \mathrm{y}''(0)$

1.Solve by the transformation $\frac{d^2y}{dt^2} + k^2y = 0$

given that y(0)=2 y'(0)=0

$$L\{y''\} + k^2L\{y\} = 0$$

$$S^2L\{y\} - sy(0) - y'(0) + K^2L\{y\} = 0$$

$$(S^2 + K^2)L\{y\} - 2S - 0 = 0$$

$$(S^2 + K^2)L\{y\} = 2S$$

$$L\{y\} = \frac{2S}{S^2 + K^2}$$

$$L^{-1}\left\{\frac{2S}{S^2 + K^2}\right\} = 2cosKt$$

2.Solve the differential equation

$y'' + 2y' + y = 6te^{-t}$ under the condition $y(0) = 0 = y'(0)$ using the Laplace transform.

$$L\{y''\} + 2L\{y'\} + L\{y\} = 6L\{te^{-t}\}$$

$$S^2L\{y\} - Sy(0) - y'(0) +$$

$$2\{SL\{y\} - y(0)\} + L\{y\} = \frac{6}{(S^2 + 1)^2}$$

$$(S^2 + 2S + 1)L\{y\} = \frac{6}{(S^2 + 1)^2}$$

$$L\{y\} = \frac{6}{(S + 1)^4}$$

$$y = L^{-1}\left\{\frac{6}{(S + 1)^4}\right\} = 6e^{-t}L^{-1}\left\{\frac{1}{S^4}\right\} = 6e^{-t}\frac{t^3}{3!}$$

$$y = e^{-t}t^3$$

3.Solve using the Laplace transform method

$y'' + 2y' - 3y = Sint$ under the conditions $y(0) = 0 = y'(0)$

$$L\{y''\} + 2L\{y'\} - 3L\{y\} = L\{Sint\}$$

$$S^2L\{y\} - Sy(0) - y'(0)$$

$$+2[SL\{Y\} - Y(0)] - 3L\{Y\} = \frac{I}{S^2 + 1}$$

$$[S^2 + 2S - 3]L\{y\} = \frac{1}{S^2 + 1}$$

$$L\{y\} = \left\{\frac{1}{(S^2 + 1)(S - 1)(S + 3)}\right\}$$

$$\frac{1}{(S^2 + 1)(S - 1)(S + 3)} = \frac{A}{S + 3} + \frac{B}{S - 1} + \frac{CS + E}{S^2 + 1}$$

$$y = -\frac{1}{40}L^{-1}\left\{\frac{1}{S + 3}\right\} + \frac{1}{8}L^{-1}\left\{\frac{1}{S - 1}\right\}$$

$$-\frac{1}{10}L^{-1}\left\{\frac{S}{S^2 + 1} - \frac{2}{S^2 + 1}\right\}$$

$$y = -\frac{1}{40}e^{-3t} + \frac{1}{8}e^{t} - \frac{1}{10}[cost - 2sint]$$

4.Solve the differential equation

$y'' + 4y' + 3y = e^{-t}$ with $y(0) = 1$ and $y'(0)$ by using Laplace transform

$$L\{y''\} + 4L\{y'\} + 3L\{y\} = L\{e^{t}\}$$

$$S^2L\{y\} - Sy(0) - y'(0) + 4\{SL[y] - y(0)\} + 3L\{y\} = \mathrm{L}\{\mathrm{e}^{-\mathrm{t}}\}$$

$$(S^2 + 4S + 3)L\{y\} - S - 1 - 4 = \frac{1}{S + 1}$$

$$(S^2 + 4S + 3)^2\{y\} = (S + 5) + \frac{1}{S + 1}$$

$$y = L^{-1}\left\{\frac{S^2 + 6S + 6}{(S + 1)(S + 1)(S + 3)}\right\}$$

$$L\{y\} = \frac{S^2 + 6S + 6}{(S - 1)^2(S + 3)}$$

$$\frac{S^2+6S+6}{(S-1)^2(S+3)}=\frac{A}{S+1}+\frac{B}{(S+1)^2}+\frac{C}{S+3}$$

$$=\frac{7}{4}\frac{1}{S+1}+\frac{1}{2(S+1)^2}+\frac{3}{4(S+3)}$$

$$y=\frac{7}{4}L^{-1}\left\{\frac{1}{S+1}\right\}+\frac{1}{2}L^{-1}\left\{\frac{1}{(S+1)^2}\right\}+\frac{3}{4}L^{-1}\left\{\frac{1}{(S+3)}\right\}$$

$$y=\frac{7}{4}L^{-1}\left\{\frac{1}{S+1}\right\}+\frac{1}{2}L^{-1}\left\{\frac{1}{(S+1)^2}\right\}$$

$$+\frac{3}{4}L^{-1}\left\{\frac{1}{(S+3)}\right\}$$

$$y=\frac{7}{4}e^{-t}+\frac{1}{2}te^{-1}-\frac{3}{4}e^{-3t}$$

5.Use Laplace transform method to solve

$\frac{d^2x}{dt^2}-\frac{2dx}{dt}+x=e^t$ with $x=2,\frac{dx}{dt}=-1\ at\ t=0$

$$L\{x''\}-2L\{x'\}+L\{x\}=L\{e^t\}$$

$$S^2L\{x\}-Sx(0)-x'(0)-2[SL(x)-x(0)]$$

$$+L\{x\}=\frac{1}{S-1}$$

$[S^2-2S+1]L\{x\}=\frac{1}{S-1}+2S-5$

$$L\{x\}=\frac{1}{(S-1)^3}+\frac{2S-5}{(S-1)^2}=\frac{2S^2-7S+6}{(S-1)^3}$$

$$=\frac{A}{S-1}+\frac{B}{(S-1)^2}+\frac{C}{(S-1)^3}$$

$$x=L^{-1}\left\{\frac{2}{S-1}\right\}-L^{-1}\left\{\frac{3}{(S-1)^2}\right\}+L^{-1}\left\{\frac{1}{(S-1)^3}\right\}$$

$$x=2e^t-3e^tt+\frac{e^tt^2}{2}$$

6.Solveby using Laplace transform $y''+4y'+3y=0$

Given that $y(0) = 0$, $y'(0) = 1$

$$S^2L\{y\} - Sy(0) - y'(0) + 4SL\{y\} - y(0) + 3L\{y\} = 0$$

$$[S^2 + 4S + 3]L\{Y\} = 1$$

$$L\{y\} = \frac{1}{S^2 + 4S + 3}$$

$$y = L^{-1}\left\{\frac{1}{S^2 + 4S + 3}\right\}$$

$$y = L^{-1}\left\{\frac{1}{(S+2)^2 - 1}\right\} = e^{-2t}sinht$$

$$= e^{-2t}L^{-1}\left\{\frac{1}{S^2 + 1}\right\}$$

7.Using Laplace transform method solve $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = e^2t$ with $y(0) = 0, y'(0) = 1$

$$L\{y''\} - 2L\{y\} + L\{y\} = L\{e^{2t}\}$$

$$S^2L\{y\} - Sy(0) - y'(0) + 2SL\{y\} - 2y(0) + L\{y\} = \frac{1}{S-2}$$

$$[S^2 + 2S + 1]L\{y\} - 1 = \frac{1}{S-1}$$

$$(S-1)^2L\{y\} = \frac{1}{S-2} + 1 = \frac{1 + S - 2}{S-2} = \frac{S-1}{S-2}$$

$$L\{y\} = \frac{(S-1)}{(S-2)(S-1)^2} = \frac{1}{(S-2)(S-2)}$$

$$L\{y\} = -\frac{1}{S-1} + \frac{1}{S-2} \quad y = L\left[-\frac{1}{S-1} + \frac{1}{S-2}\right]$$

$$y = -e^t + e^{2t} = e^{2t} - e^t$$

8.Using the Laplace transform method solve $\frac{d^2y}{dt^2}+\frac{4dy}{dx}+4y=e^{-1}$ with $y(0)=y'(0)=0$

$$L\{y''\}+4L\{y\}+4L\{y\}=L\{e^{-t}\}$$

$$S^2L\{y\}-Sy\{0\}-y'\{0\}+4SL\{y\}-4y(0)+4\mathrm{L}\{\mathrm{y}\}=\frac{1}{\mathrm{S}+1}$$

$$[S^2+4S+4]L\{4\}=\frac{1}{S+1}$$

$$L\{y\}=\frac{1}{(S+1)(S^2+4S+4)}=\frac{1}{(S+1)(S+2)^2}$$

$$y=L^{-1}\left\{\frac{1}{(S+1)(S+2)^2}\right\}$$

$$\frac{1}{(S+1)(S+2)^2}=\frac{A}{S+1}+\frac{B}{S+2}+\frac{C}{(S+2)^2}$$

$$=\frac{1}{S+1}-\frac{1}{S+2}-\frac{1}{(S+2)^2}$$

$$4=L^{-1}\left\{\frac{1}{S+1}\right\}-L^{-1}\left\{\frac{1}{S+2}\right\}-L^{-1}\left\{\frac{1}{(S+2)^2}\right\}$$

$$=e^{-t}-e^{-2t}-e^{-2t}L^{-1}\left\{\frac{1}{s^2}\right\}$$

$$=e^{-t}-e^{-2t}-e^{-2t}t$$

9.By applying laplace transform solve the differential equation $\frac{d^2y}{dt^2}+\frac{5dy}{dt}+6y=5e^{2t}$ subject to the condition $y(0)=2,\ y'(0)=1$

$$L\{y''\}+5L\{y'\}+6L\{y\}=5L\{e^{2t}\}$$

$$S^2L\{y\}-Sy(0)-y'(0)+5SL\{y\}-5y(0)+6L\{y\}=\frac{5}{S-2}$$

$$[S^2+5S+6]L\{y\}-2S-1-10=\frac{5}{S-2}$$

$$[S^2+5S+6]L\{y\}=\frac{5}{S-2}+2S+11$$

$$(S^2+5S+6)L\{y\}=\frac{5+(2S+11)(S-2)}{S-2}$$

$$L\{y\}=\frac{5+2S^2+11S-4S-22}{(S-2)(S+2)(S+3)}$$

$$L\{y\}=\frac{2S^2+7S-17}{(S-2)(S+2)(S+3)}$$

$$y=L^{-1}\left\{\frac{2S^2+7S-17}{(S-2)(S+2)(S+3)}\right\}$$

$$\frac{2S^2+7S-17}{(S-2)(S+2)(S+3)}=\frac{A}{S-2}+\frac{B}{S-3}+\frac{C}{S+2}$$

$$=\frac{23}{4}e^{2t}-4e^{-3t}+\frac{1}{4}e^{3t}$$

10.Solve the simultaneous equation using Laplace transforms $\frac{dx}{dt}+y=$ $sint$ $\frac{dy}{dt}+x=cost$ given that $x=1, y=0\ when\ t=0$

$L\{x'\}+L\{y\}=L\{sint\}$ $L\{y'\}+L\{x\}=L\{cost\}$

$$SL\{x\}-1+L\{y\}=\frac{1}{S^2+1}$$

$$SL\{y\}-y(0)+L\{x\}=\frac{S}{S^2+1}$$

$$SL\{x\}+L\{y\}=\frac{1}{S+1^2}+1\ ------1$$

$$SL\{y\}+L\{x\}=\frac{S}{S+1^2}---------2$$

subtracting 1 by 2,

$(S^2 - 1)L\{x\} = S \qquad L\{x\} = \frac{S}{S^2-1}$

$$x = L^{-1}\left\{\frac{S}{S^2-1}\right\} = cosht$$

Differentiate with respect to t.

$$\frac{dx}{dt} = sinht$$

$y = sint - sinht \quad x = cosht$

11.Solve the following differential equation using Laplace transform method

$\frac{dx}{dt} + 4y = 0 \, \frac{dy}{dt} - 9x = 0$

And $x(0) = 2 \qquad y(0) = 1$

$L\{x'\} + 4L\{y\} = 0 \qquad L\{y'\} - 9L\{x\} = 0$

$SL\{x\} + 4L\{y\} = 2 \qquad SL\{y\} - 9L\{x\} = 1$

$$SL\{x\} + 4L\{y\} = 2 - - - - - - -1 * 9$$

$$SL\{y\} - 9L\{x\} = 1 \; - - - - - - -2 * 5$$

Add equation 1 and 2 and we get

$$(S^2 + 36)L\{y\} = S + 18 \qquad L\{y\} = \frac{S+18}{S^2+36}$$

$$y = L^{-1}\left\{\frac{S}{S^2+62}\right\} + 3L^{-1}\left\{\frac{6}{S^2+62}\right\}$$

$$y = cos6t + 3sin6t$$

$$\frac{dy}{dt} = -6sin6t + 18cos6t$$

$$\frac{dy}{dt} - 9x = 0 \qquad 9x = \frac{dy}{dt}$$

$$x = \frac{1}{9}[18cos6t - 6sin6t]$$

$$x = 2cos6t - \frac{2}{3}sin6t$$

12.Solve the following equation using the Laplace transform method

$\frac{dx}{dt} - y = e^t$ $\frac{dy}{dt} + x = sint$ given $x(0) = 1$ $y(0) = 0$

$L\{x'\} - L\{y\} = L\{e^t\}$ $\qquad L\{y'\} + L\{x\} = L\{sint\}$

$SL\{x\} - x\{0\} - L\{y\} = \frac{1}{S-1}$

$$SL\{y\} - y(0) + L\{x\} = \frac{1}{S^2+2}$$

$SL\{x\} - L\{y\} = \frac{1}{S-1} + 1$ $SL\{y\} + L\{x\} = \frac{1}{S^2+1}$

$$SL\{x\} - L\{y\} = \frac{1+S-1}{S-1}$$

$$SL\{x\} - L\{y\}\frac{S}{S-1} ------1$$

$$SL\{y\} + L\{x\} = \frac{1}{S^2+1} ----2$$

Add equations 1 and 2

$$(S^2+1)L\{x\} = \frac{S^2}{S-1} + \frac{1}{(S^2+1)}$$

$$x = L^{-1}\left\{\frac{S^2}{(S-1)(S^2+1)}\right\} + L^{-1}\left\{\frac{1}{(S^2+1)^2}\right\}$$

$$\frac{S^2}{(S-1)(S^2+1)} = \frac{A}{S-1} + \frac{BS+C}{S^2+1}$$

$$= \frac{1}{2(S-1)} + \frac{S}{2(S^2+1)} + \frac{1}{2(S^2+1)}$$

$$x = \frac{1}{2}L^{-1}\left\{\frac{1}{S-1}\right\} + \frac{1}{2}\left\{\frac{S}{S+1}\right\} + L^{-1}\left\{\frac{1}{2(S^2+1)}\right\} + \frac{1}{2}L^{-1}\left[\frac{1}{(S^2+1)^2}\right]$$

$$= \frac{1}{2}e^t + \frac{1}{2}cost + \frac{1}{2}sint + \frac{1}{2}[sint - tcost]$$

$$x = \frac{1}{2}e^2 + sint + \frac{1}{2}(1-t)cost$$

$$\frac{dx}{dt} = \frac{1}{2}e^2 + cost + \frac{1}{2}(-1)cost + \frac{1}{2}(1-t)sint$$

$$\frac{dy}{dx} = \frac{1}{2}e^t + \frac{cost}{2} - \frac{1}{2}(1-t)sint$$

$$y = \frac{dx}{dt} - e^t = \frac{1}{2}e^t + \frac{cost}{2} - \frac{1}{2}(1-t)sint - e^t$$

$$y = \frac{1}{2}[at - (1-t)sint - e^t]$$

13.Solve using Laplace transform

$\frac{dx}{dt} + y = sint$ $\frac{dy}{dx} + x = cost$ under the condition $x(0) = 2$ and $y(0) = 0$

$L\{x'\} + L\{y\} = L\{sint\}$ $L\{y'\} + L\{x\} = L\{cost\}$

$SL\{x\} - x(0) + L\{y\} = \frac{1}{S^2+1}$

$$SL\{y\} - y(0) + L\{x\} = \frac{S}{S^2+1}$$

$L\{y\} + SL\{x\} = \frac{1}{S^2+1} + 2$ $SL\{y\} + L\{x\} = \frac{S}{S^2+1}$

$$SL\{x\} + L\{y\} = \frac{1}{S^2-1} + 2 \;----1$$

$$SL\{y\} + L\{x\} = \frac{S}{S^2+1} \;------2$$

Subtracting equation 1 by 2, we get

$$(S^2-1)L\{x\} = \frac{S+2S^3+2S-S}{S^2+1}$$

$$L\{x\} = \frac{2S^3+3S-S}{(S^2+1)(S^2-1)}$$

$$= \frac{2S^3+2S}{(S^2+1)(S^2-1)} = \frac{2S(S^2+1)}{(S^2+1)(S^2-1)}$$

$$L\{x\} = \frac{2S}{S^2-1} \qquad x = 2L^{-1}\left\{\frac{S}{S^2-1}\right\}$$

$$x = 2cosht \qquad \frac{dx}{dt} = 2sinht$$

$$y = sint - \frac{dx}{dt}$$

$$y = sint - 2sinht$$

14.Solve the following simultaneous differential equation by using Laplace transform $x' - 2y = cos2t$

$y' + 2x = sin2t$ given that $x(0) = 1$ and $y(0) = 0$

$$L\{x'\} - 2L\{y\} = L\{cos2t\}$$

$$L\{y'\} + 2L\{x\} = L\{sin2t\}$$

$$SL\{x\} - x(0) - 2L\{y\} = \frac{S}{S^2+4}$$

$$SL\{y\} - y(1) - 2L\{x\} = \frac{2}{S^2+4}$$

$SL\{x\} - 2L\{y\} = \frac{S}{S^2+4} + 1$-----------1

$SL\{y\} + 2L\{x\} = \frac{2}{S^2+4}$-----------2

Subtract equation 2 from 1, we get

$L\{y\} = -\frac{2}{S^2+4}$

$$y = L^{-1}\left\{\frac{2}{S^2+4}\right\}$$

$$y = -sin2t$$

$$2x = sin2t - \frac{2y}{dt}$$

$$2x = sin2t - 2cos2t$$

$$x = \frac{1}{2}sin2t - cos2t$$

15.Solve using Laplace transform $\frac{d^2y}{dt^2} - \frac{3dy}{dt} + 2y = e^{3t}$ given that $y(0) = 0$ and $y'(0) = 0$

$$L\{y''\} - 3L\{y'\} + 2L\{y\} = L\{e^{3t}\}$$

$$S^2L\{y\} - Sy(0) - y'(0) - 3[SL\{y\} - y(0)] + 2L\{y\} = \frac{1}{S-3}$$

$$[S^2 - 3S + 2]L\{y\} = \frac{1}{S-3}$$

$$L\{y\} = \frac{1}{(S-3)(S^2-3S+2)}$$

$$= \frac{1}{(S-1)(S-2)(S-3)}$$

$$y = L^{-1}\left\{\frac{1}{(S-1)(S-2)(S-3)}\right\}$$

$$= \frac{A}{S-1} + \frac{B}{S-2} + \frac{C}{S-3}$$

$$y = \frac{1}{2}L^{-1}\left\{\frac{1}{S-1}\right\} - L^{-1}\left\{\frac{1}{S-2}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{S-3}\right\}$$

$$= \frac{1}{2}e^{t} - e^{2t} + \frac{1}{2}e^{3t}$$

16.Using Laplace transform method solve $\frac{d^2y}{dt^2} + \frac{3dy}{dt} + 2y = 0$ under the condition $y(0) = 1$ $y'(0) = 0$

$$L\{y''\} + 3L\{y'\} + 2L\{y\} = 0$$

$$S^2L\{y\} - Sy(0) - y'(0) + 3[SL\{y\} - y(0)] + 2L\{y\} = 0$$

$$[S^2 + 3S + 2]L\{y\} - S - 3 = 0$$

$$L\{y\} = \frac{S+3}{S^2+3S+2} = \frac{S+3}{(S+1)(S+2)}$$

$$y = L^{-1}\left\{\frac{S+3}{(S+1)(S+2)}\right\}$$

$$= \frac{A}{S+1} + \frac{B}{S+2} = \frac{2}{S+1} - \frac{1}{S+2}$$

$$y = 2L^{-1}\left\{\frac{1}{S+1}\right\} - L^{-1}\left\{\frac{1}{S-2}\right\} = 2e^{-t} - 2e^{-2t}$$

17.Solve using the Laplace transform $y'' + 2y' + 3y = sint$ when $y(0) = 0,\ y'(0) = 0.$

$$L\{y''\} + 2L\{y'\} - 3L\{y\} = L\{sint\}$$

$S^2L\{y\} - Sy(0) - y'(0) + 2[SL(y) - y(0)] - 3L\{y\} = \frac{1}{S^2+1} \Rightarrow [S^2 + 2S - 3]L\{y\} = \frac{1}{S^2+1}$

$$y = L^{-1}\left\{\frac{1}{(S^2+1)(S-1)(S+3)}\right\}$$

$$\frac{1}{(S-1)(S+3)(S^2+1)} = \frac{A}{(S-1)} + \frac{B}{S+3} + \frac{CS+D}{S^2+1}$$

$$1 = A(S+3)(S^2+1) + B(S-1)(S^2+1) + CS + D(S-1)(S+3)$$

$S = 1,\ 1 = 8A,\ A = \frac{1}{8}$ $\quad$ $S = -3\ ,\ 1 = -40B, B = -\frac{1}{40}$

$$S = 0,\ 1 = 3A - B - 3D \Rightarrow 1 - \frac{3}{8} - \frac{1}{40} = -3D \Rightarrow$$

$$D = -\frac{1}{5}$$

$$1 = A + B + C \Rightarrow 1 - \frac{1}{8} + \frac{1}{40} = C \Rightarrow C = \frac{9}{10}$$

$$y = \frac{1}{8}L^{-1}\left\{\frac{1}{S-1}\right\} - \frac{1}{40}L^{-1}\left\{\frac{1}{S+3}\right\} + \frac{9}{10}L^{-1}\left\{\frac{S}{S^2+1}\right\} + \frac{1}{5}L^{-1}\left\{\frac{1}{S-1}\right\}$$

$$\frac{1}{8}e^t - \frac{1}{40}e^{-3t} + \frac{9}{10}cost - \frac{sint}{5}$$

18.Solve the initial value problem using the Laplace transform $(D^3 - 3D^2 + 3D - 1)y = 0$ given that $y(0) = 1$ and $y'(0) = 1$ $y''(0) = 0$

$$L\{y'''\} - 3L\{y''\} + 3L\{y'\} - L\{y\} = 0$$

$$S^3L\{y\} - S^2y(0) - Sy'(0) - y''(0) - 3[S^2L\{y\} - Sy(0) - y'(0)] + 3[SL\{y\} - y(0)] - L\{y\} = 0$$

$$[S^3 - 3S^2 + 3S - 1]L\{y\} - S^2 + 3S - 3 = 0$$

$$L\{y\} = \frac{S^2 - 3S + 3}{S^3 - 3S^2 + 3S - 1} = \frac{(S^2 - 3S + 3)}{(S-1)^3}$$

$$\frac{(S^2 - 3S + 3)}{(S-1)^3} = \frac{A}{S-1} + \frac{B}{(S-1)^2} + \frac{C}{(S-1)^3}$$

$$S^2 - 3S + 3 = A(S-1)^2 + B(S-1) + C$$

$$S = 1 \Rightarrow 1 - 3 + 3 = C \Rightarrow C = 1$$

$$A + B = 1,\ 2A + B = -3 \Rightarrow A = 2$$

$$A + B = 1{,}2 + B = 1 \Rightarrow B = -1$$

$$= \frac{2}{S-1} - \frac{1}{(S-1)^2} + \frac{1}{(S-1)^3}$$

$$y = 2L^{-1}\left\{\frac{1}{S-1}\right\} - L^{-1}\left\{\frac{1}{(S-1)^2}\right\} + L^{-1}\left\{\frac{1}{(S-1)^3}\right\}$$

$$y = 2e^t - e^t t$$